APPENDIX B

COIL PITCH AND DISTRIBUTED WINDINGS

As mentioned in Chapter 4, the induced voltage in an ac machine is sinusoidal only if the harmonic components of the air-gap flux density are suppressed. This appendix describes two techniques used by machinery designers to suppress harmonics in machines.

B.1 THE EFFECT OF COIL PITCH ON AC MACHINES

In the simple ac machine design of Section 4.4, the output voltages in the stator coils were sinusoidal because the air-gap flux density distribution was sinusoidal. If the air-gap flux density distribution had not been sinusoidal, then the output voltages in the stator would not have been sinusoidal either. They would have had the same nonsinusoidal shape as the flux density distribution.

In general, the air-gap flux density distribution in an ac machine will not be sinusoidal. Machine designers do their best to produce sinusoidal flux distributions, but of course no design is ever perfect. The actual flux distribution will consist of a fundamental sinusoidal component plus harmonics. These harmonic components of flux will generate harmonic components in the stator's voltages and currents.

The harmonic components in the stator voltages and currents are undesirable, so techniques have been developed to suppress the unwanted harmonic components in the output voltages and currents of a machine. One important technique to suppress the harmonics is the use of ***fractional-pitch windings***.

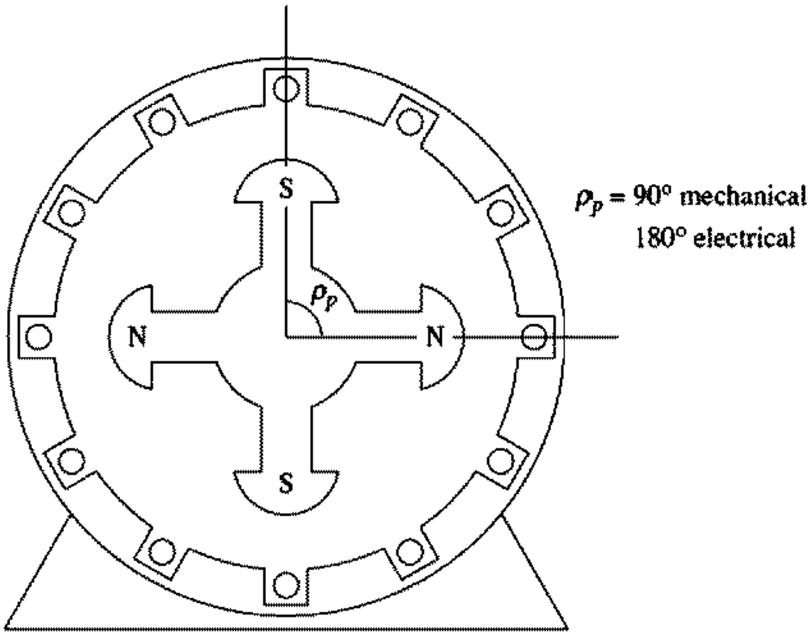


FIGURE B–1
The pole pitch of a four-pole machine is 90 mechanical or 180 electrical degrees.

The Pitch of a Coil

The *pole pitch* is the angular distance between two adjacent poles on a machine. The pole pitch of a machine in *mechanical degrees* is

$$\boxed{\rho_p = \frac{360^\circ}{P}} \tag{B–1}$$

where ρ_p is the pole pitch in *mechanical degrees* and P is the number of poles on the machine. Regardless of the number of poles on the machine, a pole pitch is always 180 *electrical degrees* (see Figure B–1).

If the stator coil stretches across the same angle as the pole pitch, it is called a *full-pitch coil*. If the stator coil stretches across an angle smaller than a pole pitch, it is called a *fractional-pitch coil*. The pitch of a fractional-pitch coil is often expressed as a fraction indicating the portion of the pole pitch it spans. For example, a 5/6-pitch coil spans five-sixths of the distance between two adjacent poles. Alternatively, the pitch of a fractional-pitch coil in electrical degrees is given by Equations (B–2):

$$\boxed{\rho = \frac{\theta_m}{\rho_p} \times 180^\circ} \tag{B–2a}$$

where θ_m is the mechanical angle covered by the coil in degrees and ρ_p is the machine's pole pitch in mechanical degrees, or

$$\boxed{\rho = \frac{\theta_m P}{2} \times 180^\circ} \tag{B–2b}$$

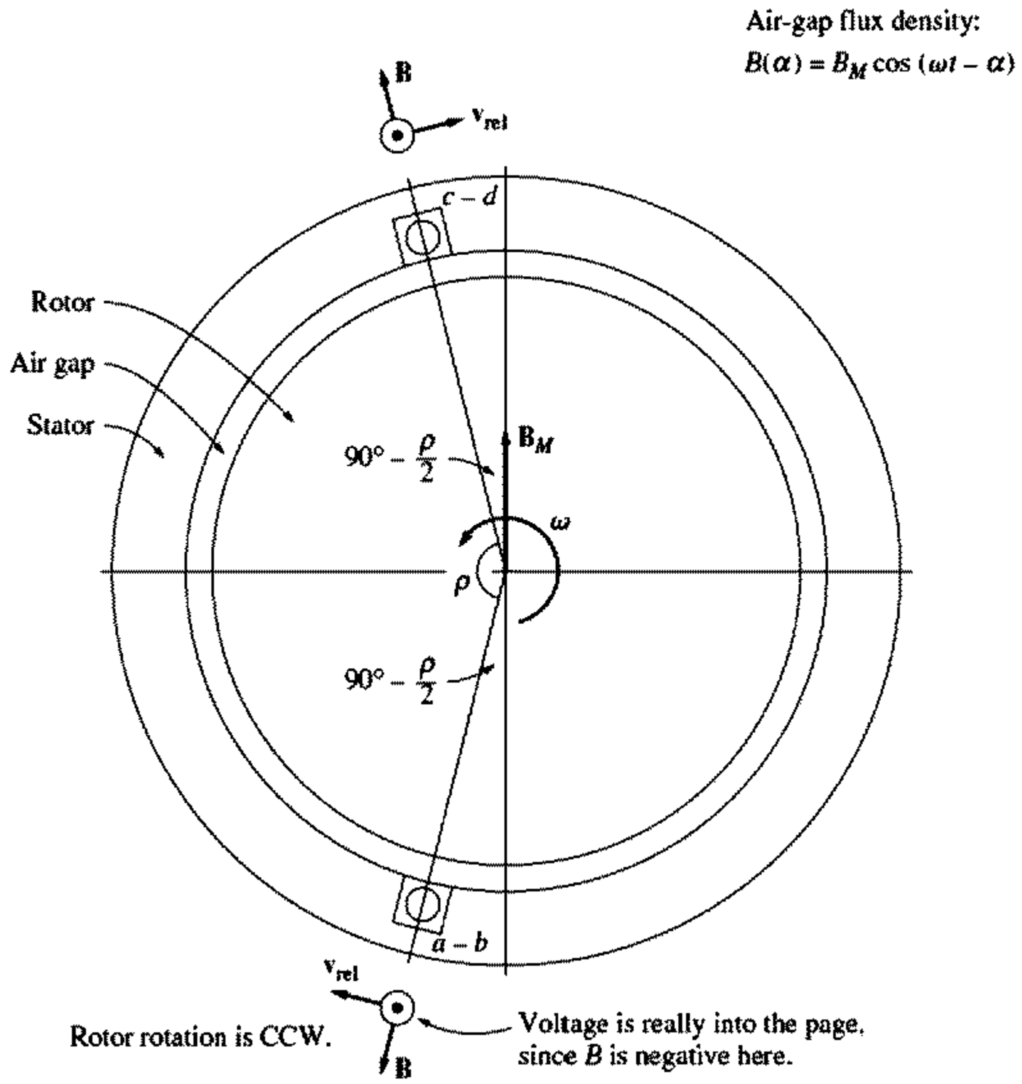


FIGURE B–2
A fractional-pitch winding of pitch ρ. The vector magnetic flux densities and velocities on the sides of the coil. The velocities are from a frame of reference in which the magnetic field is stationary.

where θ_m is the mechanical angle covered by the coil in degrees and P is the number of poles in the machine. Most practical stator coils have a fractional pitch, since a fractional-pitch winding provides some important benefits which will be explained later. Windings employing fractional-pitch coils are known as *chorded windings*.

The Induced Voltage of a Fractional-Pitch Coil

What effect does fractional pitch have on the output voltage of a coil? To find out, examine the simple two-pole machine with a fractional-pitch winding shown in Figure B–2. The pole pitch of this machine is 180°, and the coil pitch is ρ. The voltage induced in this coil by rotating the magnetic field can be found in exactly the same manner as in the previous section, by determining the voltages on each side of the coil. The total voltage will just be the sum of the voltages on the individual sides.

As before, assume that the magnitude of the flux density vector **B** in the air gap between the rotor and the stator varies sinusoidally with mechanical angle, while the direction of **B** is always radially outward. If α is the angle measured from the direction of the peak rotor flux density, then the magnitude of the flux density vector **B** at a point around the rotor is given by

$$B = B_M \cos \alpha \tag{B–3a}$$

Since the rotor is itself rotating within the stator at an angular velocity ω_m, the magnitude of the flux density vector **B** at any angle α around the *stator* is given by

$$\boxed{B = B_M \cos (\omega t - \alpha)} \tag{B–3b}$$

The equation for the induced voltage in a wire is

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \tag{1–45}$$

where $\mathbf{v}$ = velocity of the wire relative to the magnetic field
$\mathbf{B}$ = magnetic flux density vector
$\mathbf{l}$ = length of conductor in the magnetic field

This equation can only be used in a frame of reference where the magnetic field appears to be stationary. If we "sit on the magnetic field" so that the field appears to be stationary, the sides of the coil will appear to go by at an apparent velocity $\mathbf{v}_{\text{rel}}$, and the equation can be applied. Figure B–2 shows the vector magnetic field and velocities from the point of view of a stationary magnetic field and a moving wire.

1. *Segment ab.* For segment *ab* of the fractional-pitch coil, $\alpha = 90° + \rho/2$. Assuming that **B** is directed radially outward from the rotor, the angle between **v** and **B** in segment *ab* is 90°, while the quantity $\mathbf{v} \times \mathbf{B}$ is in the direction of **l**, so

$$\begin{aligned} e_{ba} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\ &= vBl \quad \text{directed out of the page} \\ &= -vB_M \cos \left[\omega_m t - \left(90° + \frac{\rho}{2}\right)\right] l \\ &= -vB_M l \cos \left(\omega_m t - 90° - \frac{\rho}{2}\right) \end{aligned} \tag{B–4}$$

   where the negative sign comes from the fact that **B** is really pointing inward when it was assumed to point outward.

2. *Segment bc.* The voltage on segment *bc* is zero, since the vector quantity $\mathbf{v} \times \mathbf{B}$ is perpendicular to **l**, so

$$e_{cb} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = 0 \tag{B–5}$$

3. *Segment cd*. For segment *cd*, the angle $\alpha = 90^\circ - \rho/2$. Assuming that **B** is directed radially outward from the rotor, the angle between **v** and **B** in segment *cd* is 90°, while the quantity $\mathbf{v} \times \mathbf{B}$ is in the direction of **l**, so

$$\begin{aligned} e_{dc} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\ &= vBl \quad \text{directed out of the page} \end{aligned}$$

$$\begin{aligned} e_{ba} &= -vB_M \cos\left[\omega_m t - \left(90^\circ - \frac{\rho}{2}\right)\right] l \\ &= -vB_M l \cos\left(\omega_m t - 90^\circ + \frac{\rho}{2}\right) \end{aligned} \tag{B–6}$$

4. *Segment da*. The voltage on segment *da* is zero, since the vector quantity $\mathbf{v} \times \mathbf{B}$ is perpendicular to **l**, so

$$e_{ad} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = 0 \tag{B–7}$$

Therefore, the total voltage on the coil will be

$$\begin{aligned} e_{\text{ind}} &= e_{ba} + e_{dc} \\ &= -vB_M l \cos\left(\omega_m t - 90^\circ - \frac{\rho}{2}\right) + vB_M l \cos\left(\omega_m t - 90^\circ + \frac{\rho}{2}\right) \end{aligned}$$

By trigonometric identities,

$$\cos\left(\omega_m t - 90^\circ - \frac{\rho}{2}\right) = \cos(\omega_m t - 90^\circ)\cos\frac{\rho}{2} + \sin(\omega_m t - 90^\circ)\sin\frac{\rho}{2}$$

$$\cos\left(\omega_m t - 90^\circ + \frac{\rho}{2}\right) = \cos(\omega_m t - 90^\circ)\cos\frac{\rho}{2} - \sin(\omega_m t - 90^\circ)\sin\frac{\rho}{2}$$

$$\sin(\omega_m t - 90^\circ) = -\cos\omega_m t$$

Therefore, the total resulting voltage is

$$\begin{aligned} e_{\text{ind}} &= vB_M l\left[-\cos(\omega_m t - 90^\circ)\cos\frac{\rho}{2} - \sin(\omega_m t - 90^\circ)\sin\frac{\rho}{2}\right. \\ &\qquad \left. + \cos(\omega_m t - 90^\circ)\cos\frac{\rho}{2} - \sin(\omega_m t - 90^\circ)\sin\frac{\rho}{2}\right] \\ &= -2vB_M l \sin\frac{\rho}{2}\sin(\omega_m t - 90^\circ) \\ &= 2vB_M l \sin\frac{\rho}{2}\cos\omega_m t \end{aligned}$$

Since $2vB_M l$ is equal to $\phi\omega$, the final expression for the voltage in a single turn is

$$\boxed{e_{\text{ind}} = \phi\omega \sin\frac{\rho}{2}\cos\omega_m t} \tag{B–8}$$

This is the same value as the voltage in a full-pitch winding except for the sin $\rho/2$ term. It is customary to define this term as the *pitch factor* k_p of the coil. The pitch factor of a coil is given by

$$\boxed{k_p = \sin\frac{\rho}{2}} \tag{B–9}$$

In terms of the pitch factor, the induced voltage on a single-turn coil is

$$e_{\text{ind}} = k_p \phi\omega \cos \omega_m t \tag{B–10}$$

The total voltage in an N-turn fractional-pitch coil is thus

$$e_{\text{ind}} = N_C\, k_p \phi\omega \cos \omega_m t \tag{B–11}$$

and its peak voltage is

$$E_{\max} = N_C\, k_p \phi\omega \tag{B–12}$$

$$= 2\pi N_C\, k_p \phi f \tag{B–13}$$

Therefore, the rms voltage of any phase of this three-phase stator is

$$E_A = \frac{2\pi}{\sqrt{2}} N_C k_p \phi f \tag{B–14}$$

$$= \sqrt{2}\pi N_C k_p \phi f \tag{B–15}$$

Note that for a full-pitch coil, $\rho = 180°$ and Equation (B–15) reduces to the same result as before.

For machines with more than two poles, Equation (B–9) gives the pitch factor if the coil pitch p is in electrical degrees. If the coil pitch is given in mechanical degrees, then the pitch factor can be given by

$$\boxed{k_p = \sin\frac{\theta_m P}{2}} \tag{B–16}$$

Harmonic Problems and Fractional-Pitch Windings

There is a very good reason for using fractional-pitch windings. It concerns the effect of the nonsinusoidal flux density distribution in real machines. This problem can be understood by examining the machine shown in Figure B–3. This figure shows a salient-pole synchronous machine whose rotor is sweeping across the stator surface. Because the reluctance of the magnetic field path is *much lower* directly under the center of the rotor than it is toward the sides (smaller air gap), the flux is strongly concentrated at that point and the flux density is very high there. The resulting induced voltage in the winding is shown in Figure B–3. *Notice that it is not sinusoidal—it contains many harmonic frequency components.*

Because the resulting voltage waveform is symmetric about the center of the rotor flux, no *even harmonics* are present in the phase voltage. However, all

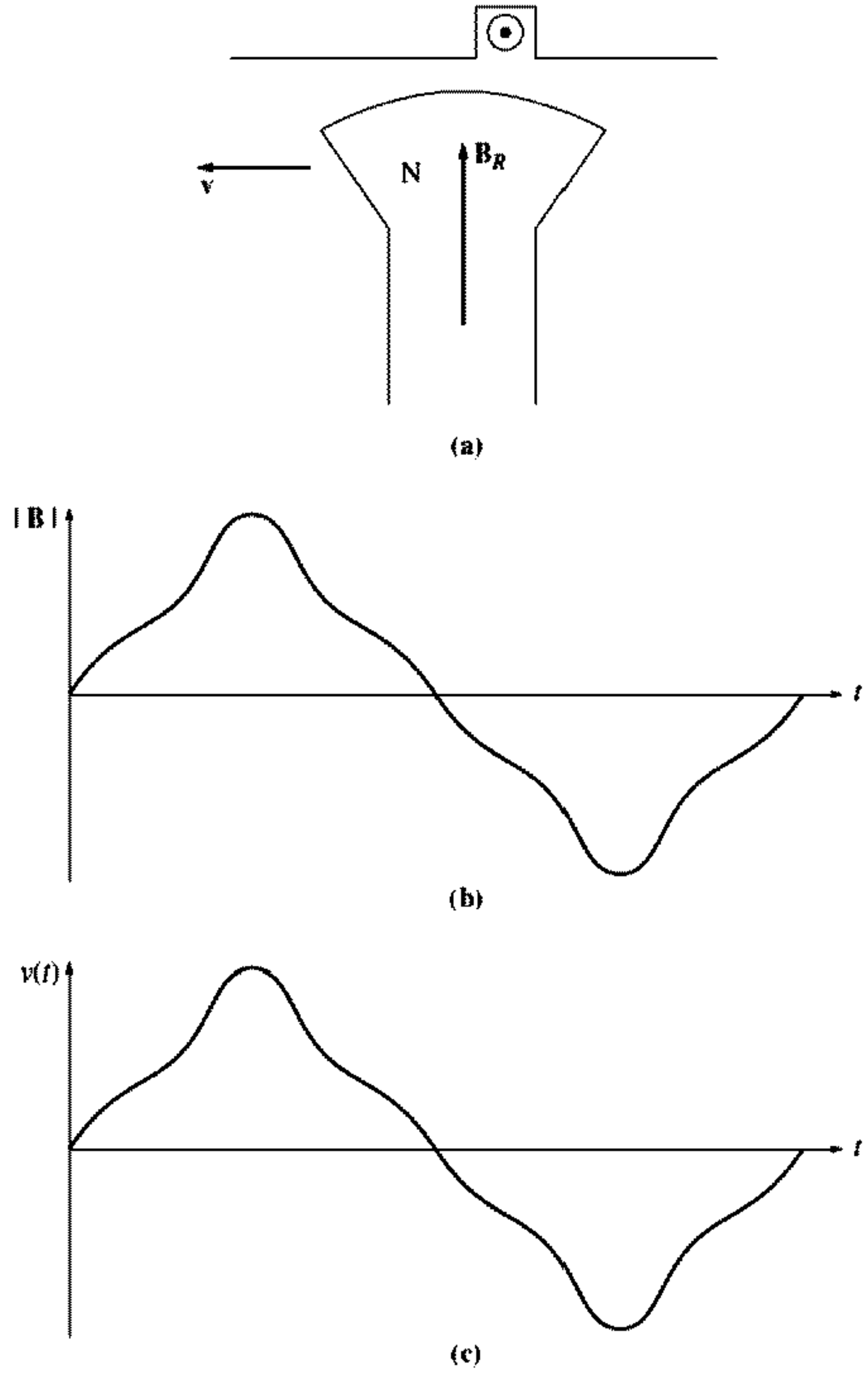


FIGURE B–3
(a) A ferromagnetic rotor sweeping past a stator conductor. (b) The flux density distribution of the magnetic field as a function of time at a point on the stator surface. (c) The resulting induced voltage in the conductor. Note that the voltage is directly proportional to the magnetic flux density at any given time.

the odd harmonics (third, fifth, seventh, ninth, etc.) *are* present in the phase voltage to some extent and need to be dealt with in the design of ac machines. In general, the higher the number of a given harmonic frequency component, the lower its magnitude in the phase output voltage; so beyond a certain point (above the ninth harmonic or so) the effects of higher harmonics may be ignored.

When the three phases are Y or Δ connected, some of the harmonics disappear from the output of the machine as a result of the three-phase connection. The third-harmonic component is one of these. If the fundamental voltages in each of the three phases are given by

$$e_a(t) = E_{M1} \sin \omega t \qquad \text{V} \tag{B–17a}$$

$$e_b(t) = E_{M1} \sin (\omega t - 120°) \qquad \text{V} \tag{B–17b}$$

$$e_c(t) = E_{M1} \sin (\omega t - 240°) \qquad \text{V} \tag{B–17c}$$

then the third-harmonic components of voltage will be given by

$$e_{a3}(t) = E_{M3} \sin 3\omega t \qquad \text{V} \tag{B–18a}$$

$$e_{b3}(t) = E_{M3} \sin (3\omega t - 360°) \qquad \text{V} \tag{B–18b}$$

$$e_{c3}(t) = E_{M3} \sin (3\omega t - 720°) \qquad \text{V} \tag{B–18c}$$

Notice that *the third-harmonic components of voltage are all identical* in each phase. If the synchronous machine is Y-connected, then the third-harmonic voltage *between any two terminals* will be zero (even though there may be a large third-harmonic component of voltage in each phase). If the machine is Δ-connected, then the three third-harmonic components all add and drive a third-harmonic current around inside the Δ-winding of the machine. Since the third-harmonic voltages are dropped across the machine's internal impedances, there is again no significant third-harmonic component of voltage at the terminals.

This result applies not only to third-harmonic components but also to any *multiple* of a third-harmonic component (such as the ninth harmonic). Such special harmonic frequencies are called *triplen harmonics* and are automatically suppressed in three-phase machines.

The remaining harmonic frequencies are the fifth, seventh, eleventh, thirteenth, etc. Since the strength of the harmonic components of voltage decreases with increasing frequency, most of the actual distortion in the sinusoidal output of a synchronous machine is caused by the fifth and seventh harmonic frequencies, sometimes called the *belt harmonics*. If a way could be found to reduce these components, then the machine's output voltage would be essentially a pure sinusoid at the fundamental frequency (50 or 60 Hz).

How can some of the harmonic content of the winding's terminal voltage be eliminated?

One way is to design the rotor itself to distribute the flux in an approximately sinusoidal shape. Although this action will help reduce the harmonic content of the output voltage, it may not go far enough in that direction. An additional step that is used is to design the machine with fractional-pitch windings.

The key to the effect of fractional-pitch windings on the voltage produced in a machine's stator is that the electrical angle of the nth harmonic is n times the electrical angle of the fundamental frequency component. In other words, if a coil spans 150 electrical degrees at its fundamental frequency, it will span 300 electrical degrees at its second-harmonic frequency, 450 electrical degrees at its third-harmonic frequency, and so forth. If ρ represents the electrical angle spanned by the coil at its *fundamental* frequency and ν is the number of the harmonic being examined, then the coil will span $\nu\rho$ electrical degrees at that harmonic frequency. Therefore, the pitch factor of the coil at the harmonic frequency can be expressed as

$$\boxed{k_p = \sin\frac{\nu\rho}{2}} \qquad \text{(B–19)}$$

The important consideration here is that ***the pitch factor of a winding is different for each harmonic frequency.*** By a proper choice of coil pitch it is possible to almost eliminate harmonic frequency components in the output of the machine. We can now see how harmonics are suppressed by looking at a simple example problem.

Example B–1. A three-phase, two-pole stator has coils with a 5/6 pitch. What are the pitch factors for the harmonics present in this machine's coils? Does this pitch help suppress the harmonic content of the generated voltage?

Solution

The pole pitch in mechanical degrees of this machine is

$$\rho_p = \frac{360°}{P} = 180° \qquad \text{(B–1)}$$

Therefore, the mechanical pitch angle of these coils is five-sixths of 180°, or 150°. From Equation (B–2a), the resulting pitch in electrical degrees is

$$\rho = \frac{\theta_m}{\rho_p} \times 180° = \frac{150°}{180°} \times 180° = 150° \qquad \text{(B–2a)}$$

The mechanical pitch angle is equal to the electrical pitch angle only because this is a two-pole machine. For any other number of poles, they would not be the same.

Therefore, the pitch factors for the fundamental and the higher odd harmonic frequencies (remember, the even harmonics are already gone) are

Fundamental: $k_p = \sin\frac{150°}{2} = 0.966$

Third harmonic: $k_p = \sin\frac{3(150°)}{2} = -0.707$ (This is a triplen harmonic not present in the three-phase output.)

Fifth harmonic: $k_p = \sin\frac{5(150°)}{2} = 0.259$

Seventh harmonic: $k_p = \sin\frac{7(150°)}{2} = 0.259$

Ninth harmonic: $k_p = \sin\frac{9(150°)}{2} = -0.707$ (This is a triplen harmonic not present in the three-phase output.)

The third- and ninth-harmonic components are suppressed only slightly by this coil pitch, but that is unimportant since they do not appear at the machine's terminals anyway. Between the effects of triplen harmonics and the effects of the coil pitch, the *third, fifth, seventh, and ninth harmonics are suppressed relative to the fundamental frequency.* Therefore, employing fractional-pitch windings will drastically reduce the harmonic content of the machine's output voltage while causing only a small decrease in its fundamental voltage.

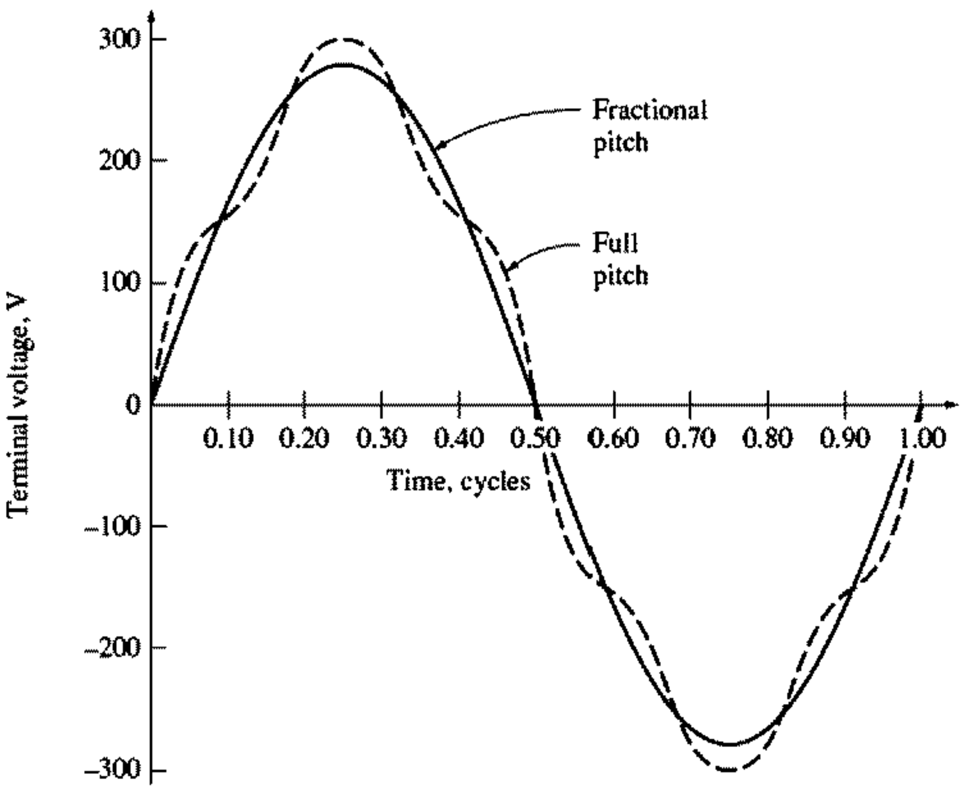


FIGURE B–4
The line voltage out of a three-phase generator with full-pitch and fractional-pitch windings. Although the peak voltage of the fractional-pitch winding is slightly smaller than that of the full-pitch winding, its output voltage is much purer.

The terminal voltage of a synchronous machine is shown in Figure B–4 both for full-pitch windings and for windings with a pitch $\rho = 150°$. Notice that the fractional-pitch windings produce a large visible improvement in waveform quality.

It should be noted that there are certain types of higher-frequency harmonics, called *tooth* or *slot harmonics*, which cannot be suppressed by varying the pitch of stator coils. These slot harmonics will be discussed in conjunction with distributed windings in Section B.2.

B.2 DISTRIBUTED WINDINGS IN AC MACHINES

In the previous section, the windings associated with each phase of an ac machine were implicitly assumed to be concentrated in a single pair of slots on the stator surface. In fact, the windings associated with each phase are almost always distributed among several adjacent pairs of slots, because it is simply impossible to put all the conductors into a single slot.

The construction of the stator windings in real ac machines is quite complicated. Normal ac machine stators consist of several coils in each phase, distributed in slots around the inner surface of the stator. In larger machines, each coil is a preformed unit consisting of a number of turns, each turn insulated from the others and from the side of the stator itself (see Figure B–5). The voltage in any

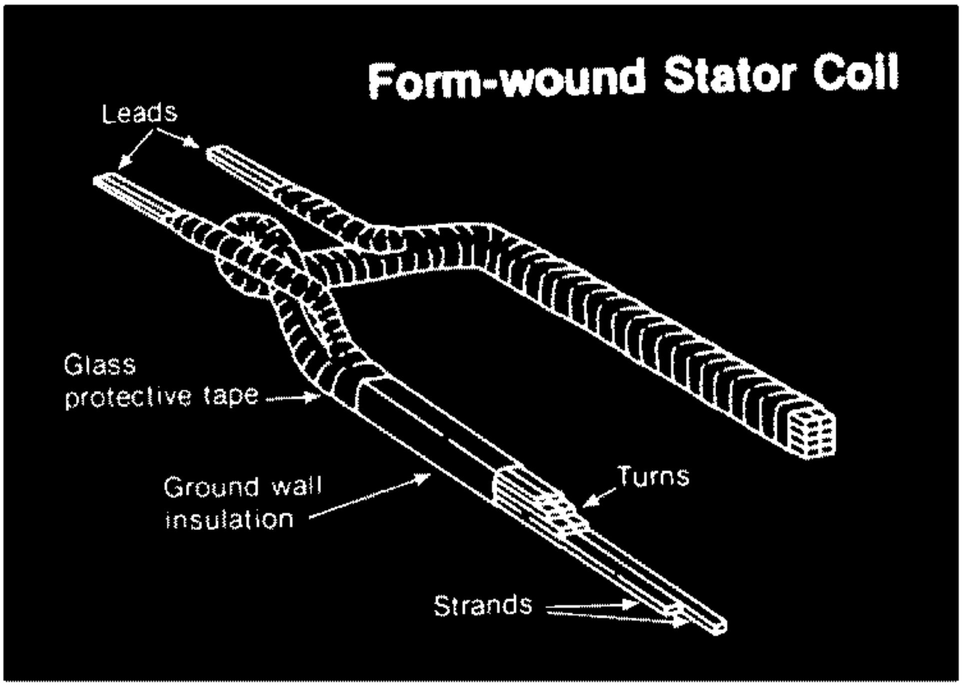
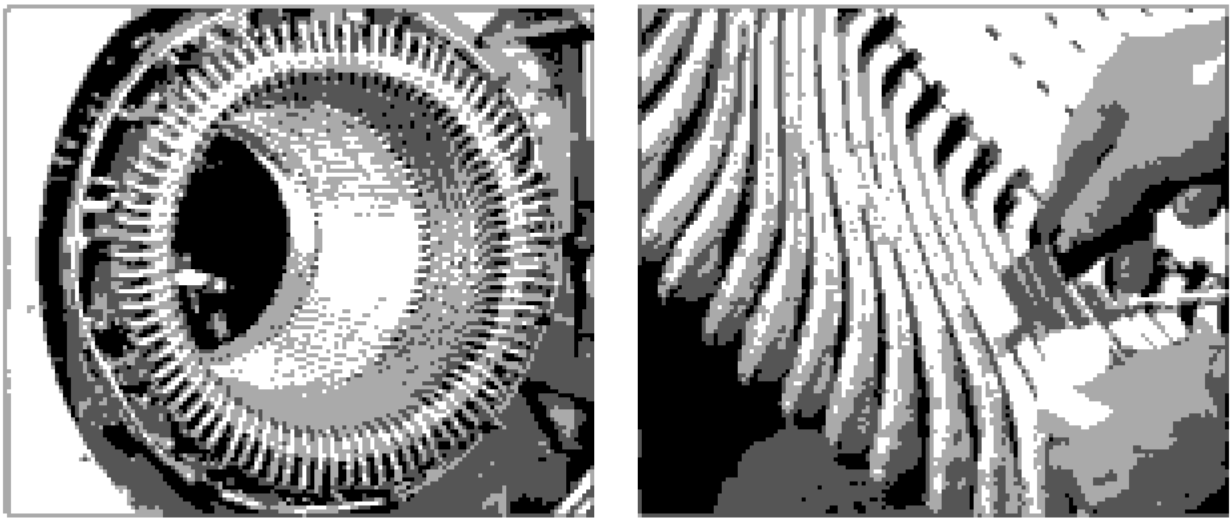


FIGURE B–5
A typical preformed stator coil. (*Courtesy of General Electric Company.*)



(a) (b)

FIGURE B–6
(a) An ac machine stator with preformed stator coils. (*Courtesy of Westinghouse Electric Company.*) (b) A close-up view of the coil ends on a stator. Note that one side of the coil will be outermost in its slot and the other side will be innermost in its slot. This shape permits a single standard coil form to be used for every slot on the stator. (*Courtesy of General Electric Company.*)

single turn of wire is very small, and it is only by placing many of these turns in series that reasonable voltages can be produced. The large number of turns is normally physically divided among several coils, and the coils are placed in slots equally spaced along the surface of the stator, as shown in Figure B–6.

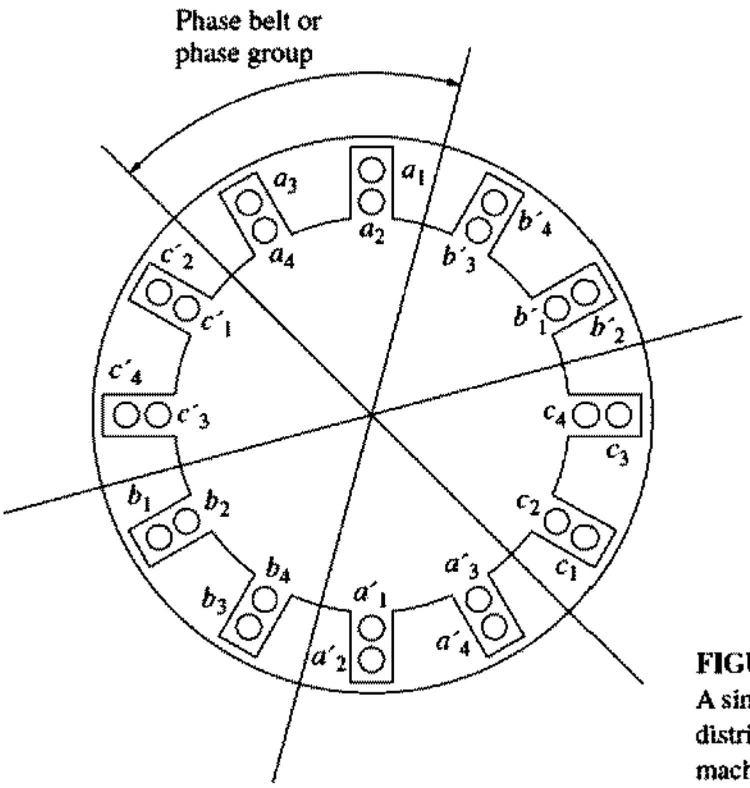


FIGURE B–7
A simple double-layer full-pitch distributed winding for a two-pole ac machine.

The spacing in degrees between adjacent slots on a stator is called the *slot pitch* γ of the stator. The slot pitch can be expressed in either mechanical or electrical degrees.

Except in very small machines, stator coils are normally formed into *double-layer windings*, as shown in Figure B–7. Double-layer windings are usually easier to manufacture (fewer slots for a given number of coils) and have simpler end connections than single-layer windings. They are therefore much less expensive to build.

Figure B–7 shows a distributed full-pitch winding for a two-pole machine. In this winding, there are four coils associated with each phase. All the coil sides of a given phase are placed in adjacent slots, and these sides are known as a *phase belt* or *phase group*. Notice that there are six phase belts on this two-pole stator. In general, there are $3P$ phase belts on a P-pole stator, P of them in each phase.

Figure B–8 shows a distributed winding using fractional-pitch coils. Notice that this winding still has phase belts, but that the phases of coils within an individual slot may be mixed. The pitch of the coils is 5/6 or 150 electrical degrees.

The Breadth or Distribution Factor

Dividing the total required number of turns into separate coils permits more efficient use of the inner surface of the stator, and it provides greater structural strength, since the slots carved in the frame of the stator can be smaller. However, the fact that the turns composing a given phase lie at different angles means that their voltages will be somewhat smaller than would otherwise be expected.

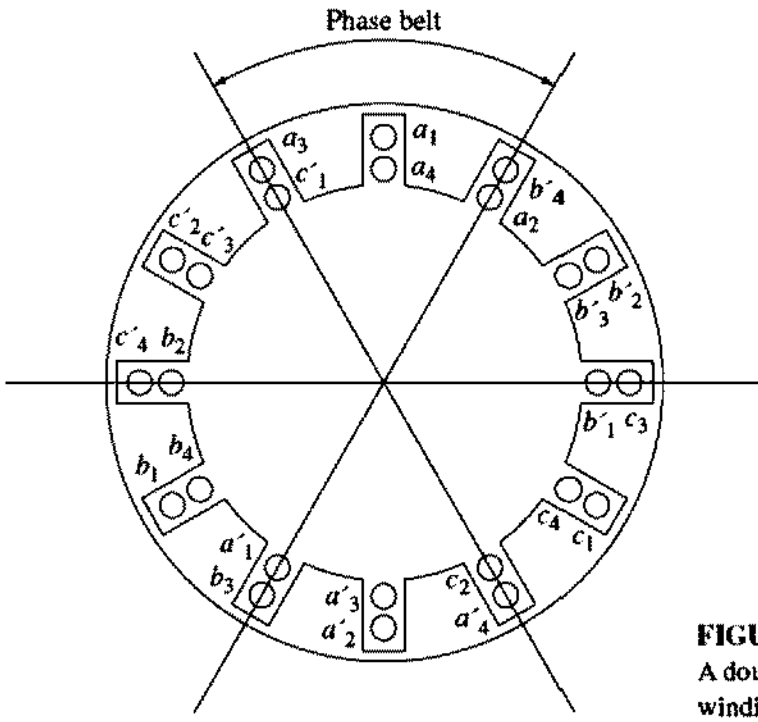


FIGURE B–8
A double-layer fractional-pitch ac winding for a two-pole ac machine.

To illustrate this problem, examine the machine shown in Figure B–9. This machine has a single-layer winding, with the stator winding of each phase (each phase belt) distributed among three slots spaced 20° apart.

If the central coil of phase a initially has a voltage given by

$$\mathbf{E}_{a2} = E \angle 0^\circ \text{ V}$$

then the voltages in the other two coils in phase a will be

$$\mathbf{E}_{a1} = E \angle -20^\circ \text{ V}$$

$$\mathbf{E}_{a3} = E \angle 20^\circ \text{ V}$$

The total voltage in phase a is given by

$$\begin{aligned}\mathbf{E}_a &= \mathbf{E}_{a1} + \mathbf{E}_{a2} + \mathbf{E}_{a3} \\ &= E \angle -20^\circ + E \angle 0^\circ + E \angle 20^\circ \\ &= E \cos(-20^\circ) + jE \sin(-20^\circ) + E + E \cos 20^\circ + jE \sin 20^\circ \\ &= E + 2E \cos 20^\circ = 2.879\, E\end{aligned}$$

This voltage in phase a is not quite what would have been expected if the coils in a given phase had all been concentrated in the same slot. Then, the voltage E_a would have been equal to $3E$ instead of $2.879E$. The ratio of the actual voltage in a phase of a distributed winding to its expected value in a concentrated winding with the same number of turns is called the *breadth factor* or *distribution factor* of winding. The distribution factor is defined as

$$k_d = \frac{V_\phi \text{ actual}}{V_\phi \text{ expected with no distribution}} \tag{B–20}$$

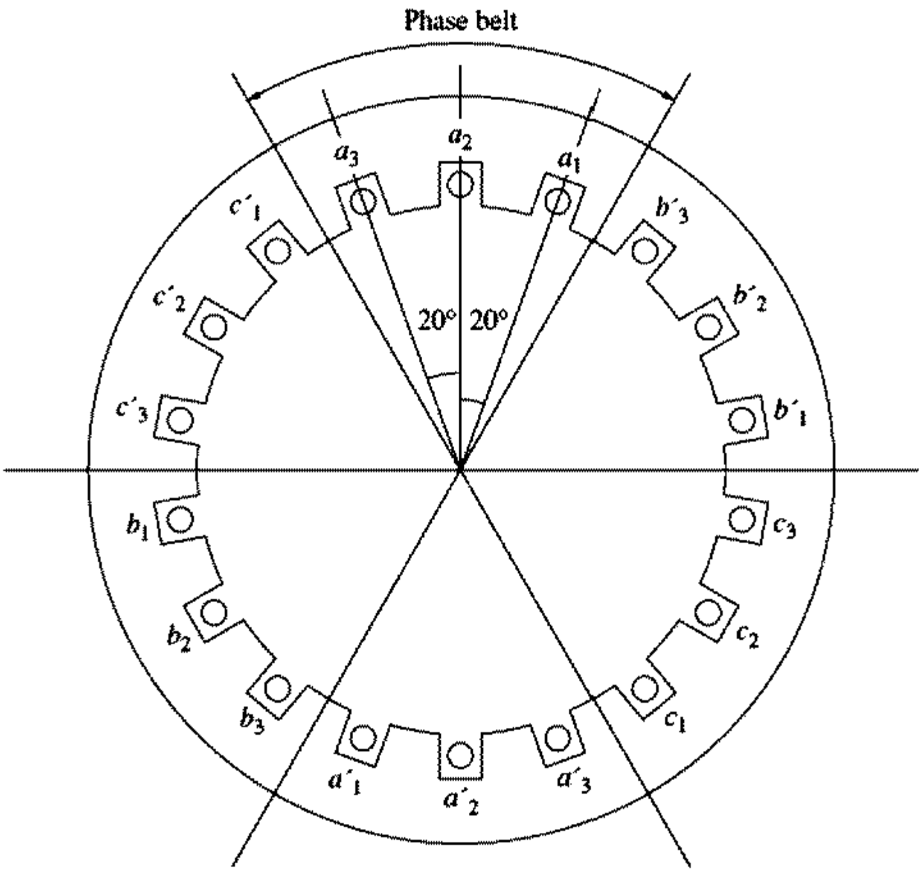


FIGURE B–9
A two-pole stator with a single-layer winding consisting of three coils per phase, each separated by 20°.

The distribution factor for the machine in Figure B–9 is thus

$$k_d = \frac{2.879E}{3E} = 0.960 \tag{B–21}$$

The distribution factor is a convenient way to summarize the decrease in voltage caused by the spatial distribution of the coils in a stator winding.

It can be shown (see Reference 1, page 726) that, for a winding with n slots per phase belt spaced γ degrees apart, the distribution factor is given by

$$\boxed{k_d = \frac{\sin(n\gamma/2)}{n \sin(\gamma/2)}} \tag{B–22}$$

Notice that for the previous example with $n = 3$ and $\gamma = 20°$, the distribution factor becomes

$$k_d = \frac{\sin(n\gamma/2)}{n \sin(\gamma/2)} = \frac{\sin[(3)(20°)/2]}{3 \sin(20°/2)} = 0.960 \tag{B–22}$$

which is the same result as before.

The Generated Voltage Including Distribution Effects

The rms voltage in a single coil of N_C turns and pitch factor k_p was previously determined to be

$$E_A = \sqrt{2}\pi N_C k_p \phi f \tag{B–15}$$

If a stator phase consists of i coils, each containing N_C turns, then a total of $N_P = iN_C$ turns will be present in the phase. The voltage present across the phase will just be the voltage due to N_P turns all in the same slot times the reduction caused by the distribution factor, so the total phase voltage will become

$$\boxed{E_A = \sqrt{2}\pi N_P k_p k_d \phi f} \tag{B–23}$$

The pitch factor and the distribution factor of a winding are sometimes combined for ease of use into a single *winding factor* k_w. The winding factor of a stator is given by

$$\boxed{k_w = k_p k_d} \tag{B–24}$$

Applying this definition to the equation for the voltage in a phase yields

$$\boxed{E_A = \sqrt{2}\pi N_P k_w \phi f} \tag{B–25}$$

Example B–2. A simple two-pole, three-phase, Y-connected synchronous machine stator is used to make a generator. It has a double-layer coil construction, with four stator coils per phase distributed as shown in Figure B–8. Each coil consists of 10 turns. The windings have an electrical pitch of 150°, as shown. The rotor (and the magnetic field) is rotating at 3000 r/min, and the flux per pole in this machine is 0.019 Wb.

(a) What is the slot pitch of this stator in mechanical degrees? In electrical degrees?
(b) How many slots do the coils of this stator span?
(c) What is the magnitude of the phase voltage of one phase of this machine's stator?
(d) What is the machine's terminal voltage?
(e) How much suppression does the fractional-pitch winding give for the fifth-harmonic component of the voltage relative to the decrease in its fundamental component?

Solution

(a) This stator has 6 phase belts with 2 slots per belt, so it has a total of 12 slots. Since the entire stator spans 360°, the slot pitch of this stator is

$$\gamma = \frac{360°}{12} = 30°$$

This is both its electrical and mechanical pitch, since this is a two-pole machine.

(b) Since there are 12 slots and 2 poles on this stator, there are 6 slots per pole. A coil pitch of 150 electrical degrees is 150°/180° = 5/6, so the coils must span 5 stator slots.

(c) The frequency of this machine is

$$f = \frac{n_m P}{120} = \frac{(3000 \text{ r/min})(2 \text{ poles})}{120} = 50 \text{ Hz}$$

From Equation (B–19), the pitch factor for the fundamental component of the voltage is

$$k_p = \sin\frac{\nu\rho}{2} = \sin\frac{(1)(150°)}{2} = 0.966 \tag{B–19}$$

Although the windings in a given phase belt are in three slots, the two outer slots have only one coil each from the phase. Therefore, the winding essentially occupies two complete slots. The winding distribution factor is

$$k_d = \frac{\sin(n\gamma/2)}{n\sin(\gamma/2)} = \frac{\sin[(2)(30°)/2]}{2\sin(30°/2)} = 0.966 \tag{B–22}$$

Therefore, the voltage in a single phase of this stator is

$$\begin{aligned} E_A &= \sqrt{2}\ \pi N_p k_p k_d \phi f \\ &= \sqrt{2}\ \pi(40 \text{ turns})(0.966)(0.966)(0.019 \text{ Wb})(50 \text{ Hz}) \\ &= 157 \text{ V} \end{aligned}$$

(d) This machine's terminal voltage is

$$V_T = \sqrt{3}E_A = \sqrt{3}(157 \text{ V}) = 272 \text{ V}$$

(e) The pitch factor for the fifth-harmonic component is

$$k_p = \sin\frac{\nu\rho}{2} = \sin\frac{(5)(150°)}{2} = 0.259 \tag{B–19}$$

Since the pitch factor of the fundamental component of the voltage was 0.966 and the pitch factor of the fifth-harmonic component of voltage is 0.259, the fundamental component was decreased 3.4 percent, while the fifth-harmonic component was decreased 74.1 percent. Therefore, the fifth-harmonic component of the voltage is decreased 70.7 percent more than the fundamental component is.

Tooth or Slot Harmonics

Although distributed windings offer advantages over concentrated windings in terms of stator strength, utilization, and ease of construction, the use of distributed windings introduces an additional problem into the machine's design. The presence of uniform slots around the inside of the stator causes regular variations in reluctance and flux along the stator's surface. These regular variations produce harmonic components of voltage called *tooth* or *slot harmonics* (see Figure B–10). Slot harmonics occur at frequencies set by the spacing between adjacent slots and are given by

$$\boxed{\nu_{\text{slot}} = \frac{2MS}{P} \pm 1} \tag{B–26}$$

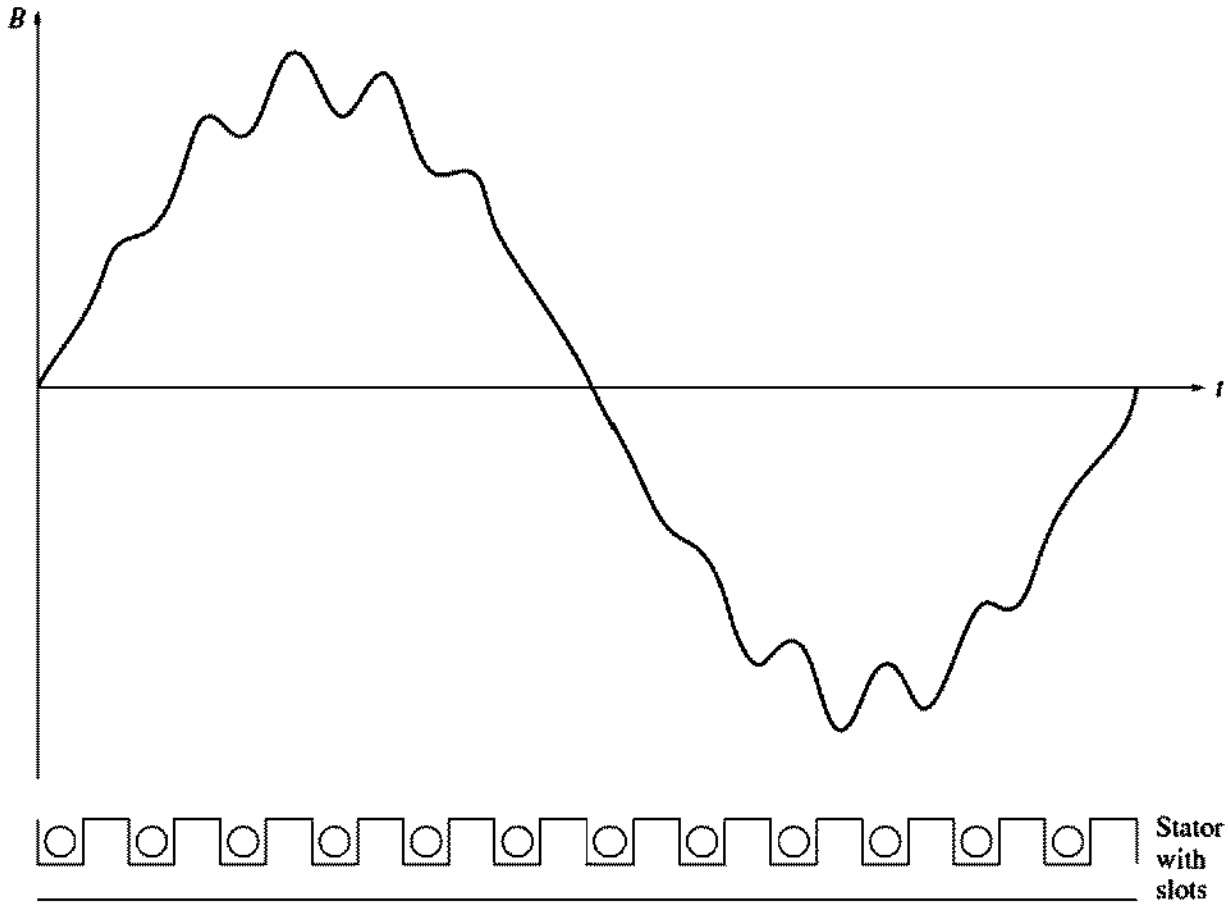


FIGURE B–10
Flux density variations in the air gap due to the tooth or slot harmonics. The reluctance of each slot is higher than the reluctance of the metal surface between the slots, so flux densities are lower directly over the slots.

where ν_{slot} = number of the harmonic component
S = number of slots on stator
M = an integer
P = number of poles on machine

The value $M = 1$ yields the lowest-frequency slot harmonics, which are also the most troublesome ones.

Since these harmonic components are set by the spacing *between adjacent coil slots*, variations in coil pitch and distribution cannot reduce these effects. Regardless of a coil's pitch, it *must* begin and end in a slot, and therefore the coil's spacing is an integral multiple of the basic spacing causing slot harmonics in the first place.

For example, consider a 72-slot, six-pole ac machine stator. In such a machine, the two lowest and most troublesome stator harmonics are

$$\nu_{\text{slot}} = \frac{2MS}{P} \pm 1 \qquad \text{(B–26)}$$

$$= \frac{2(1)(72)}{6} \pm 1 = 23, 25$$

These harmonics are at 1380 and 1500 Hz in a 60-Hz machine.

Slot harmonics cause several problems in ac machines:

1. They induce harmonics in the generated voltage of ac generators.
2. The interaction of stator and rotor slot harmonics produces parasitic torques in induction motors. These torques can seriously affect the shape of the motor's torque-speed curve.
3. They introduce vibration and noise in the machine.
4. They increase core losses by introducing high-frequency components of voltages and currents into the teeth of the stator.

Slot harmonics are especially troublesome in induction motors, where they can induce harmonics of the same frequency into the rotor field circuit, further reinforcing their effects on the machine's torque.

Two common approaches are taken in reducing slot harmonics. They are *fractional-slot windings* and *skewed rotor conductors.*

Fractional-slot windings involve using a fractional number of slots per rotor pole. All previous examples of distributed windings have been integral-slot windings: i.e., they have had 2, 3, 4, or some other integral number of slots per pole. On the other hand, a fractional-slot stator might be constructed with 2½ slots per pole. The offset between adjacent poles provided by fractional-slot windings helps to reduce both belt and slot harmonics. This approach to reducing harmonics may be used on any type of ac machine. Fractional-slot harmonics are explained in detail in References 1 and 2.

The other, much more common, approach to reducing slot harmonics is *skewing* the conductors on the rotor of the machine. This approach is primarily used on induction motors. The conductors on an induction motor rotor are given a slight twist, so that when one end of a conductor is under one stator slot, the other end of the coil is under a neighboring slot. This rotor construction is shown in Figure B–11. Since a single rotor conductor stretches from one coil slot to the next (a distance corresponding to one full electrical cycle of the lowest slot harmonic frequency), the voltage components due to the slot harmonic variations in flux cancel.

B.3 SUMMARY

In real machines, the stator coils are often of fractional pitch, meaning that they do not reach completely from one magnetic pole to the next. Making the stator windings fractional-pitch reduces the magnitude of the output voltage slightly, but at the same time attenuates the harmonic components of voltage drastically, resulting in a much smoother output voltage from the machine. A stator winding using fractional-pitch coils is often called a *chorded winding.*

Certain higher-frequency harmonics, called tooth or slot harmonics, cannot be suppressed with fractional-pitch coils. These harmonics are especially trouble-

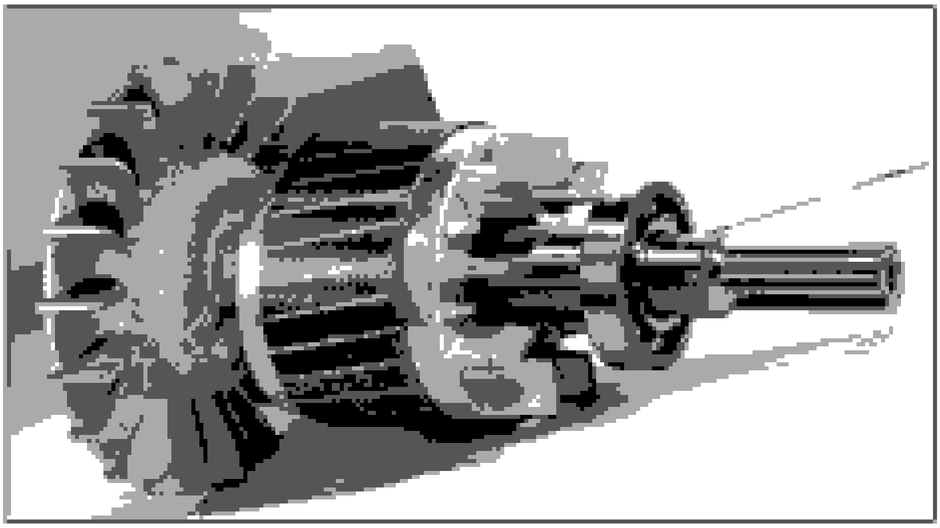


FIGURE B–11
An induction motor rotor exhibiting conductor skewing. The skew of the rotor conductors is just equal to the distance between one stator slot and the next one. (*Courtesy of MagneTek, Inc.*)

some in induction motors. They can be reduced by employing fractional-slot windings or by skewing the rotor conductors of an induction motor.

Real ac machine stators do not simply have one coil for each phase. In order to get reasonable voltages out of a machine, several coils must be used, each with a large number of turns. This fact requires that the windings be distributed over some range on the stator surface. Distributing the stator windings in a phase reduces the possible output voltage by the distribution factor k_d, but it makes it physically easier to put more windings on the machine.

QUESTIONS

B–1. Why are distributed windings used instead of concentrated windings in ac machine stators?

B–2. (*a*) What is the distribution factor of a stator winding? (*b*) What is the value of the distribution factor in a concentrated stator winding?

B–3. What are chorded windings? Why are they used in an ac stator winding?

B–4. What is pitch? What is the pitch factor? How are they related to each other?

B–5. Why are third-harmonic components of voltage not found in three-phase ac machine outputs?

B–6. What are triplen harmonics?

B–7. What are slot harmonics? How can they be reduced?

B–8. How can the magnetomotive force (and flux) distribution in an ac machine be made more nearly sinusoidal?

PROBLEMS

B–1. A two-slot three-phase stator armature is wound for two-pole operation. If fractional-pitch windings are to be used, what is the best possible choice for winding pitch if it is desired to eliminate the fifth-harmonic component of voltage?

B–2. Derive the relationship for the winding distribution factor k_d in Equation (B–22).

B–3. A three-phase four-pole synchronous machine has 96 stator slots. The slots contain a double-layer winding (two coils per slot) with four turns per coil. The coil pitch is 19/24.

(a) Find the slot and coil pitch in electrical degrees.

(b) Find the pitch, distribution, and winding factors for this machine.

(c) How well will this winding suppress third, fifth, seventh, ninth, and eleventh harmonics? Be sure to consider the effects of both coil pitch and winding distribution in your answer.

B–4. A three-phase four-pole winding of the double-layer type is to be installed on a 48-slot stator. The pitch of the stator windings is 5/6, and there are 10 turns per coil in the windings. All coils in each phase are connected in series, and the three phases are connected in Δ. The flux per pole in the machine is 0.054 Wb, and the speed of rotation of the magnetic field is 1800 r/min.

(a) What is the pitch factor of this winding?

(b) What is the distribution factor of this winding?

(c) What is the frequency of the voltage produced in this winding?

(d) What are the resulting phase and terminal voltages of this stator?

B–5. A three-phase, Y-connected, six-pole synchronous generator has six slots per pole on its stator winding. The winding itself is a chorded (fractional-pitch) double-layer winding with eight turns per coil. The distribution factor $k_d = 0.956$, and the pitch factor $k_p = 0.981$. The flux in the generator is 0.02 Wb per pole, and the speed of rotation is 1200 r/min. What is the line voltage produced by this generator at these conditions?

B–6. A three-phase, Y-connected, 50-Hz, two-pole synchronous machine has a stator with 18 slots. Its coils form a double-layer chorded winding (two coils per slot), and each coil has 60 turns. The pitch of the stator coils is 8/9.

(a) What rotor flux would be required to produce a terminal (line-to-line) voltage of 6 kV?

(b) How effective are coils of this pitch at reducing the fifth-harmonic component of voltage? The seventh-harmonic component of voltage?

B–7. What coil pitch could be used to completely eliminate the seventh-harmonic component of voltage in ac machine armature (stator)? What is the *minimum* number of slots needed on an eight-pole winding to exactly achieve this pitch? What would this pitch do to the fifth-harmonic component of voltage?

B–8. A 13.8-kV, Y-connected, 60-Hz, 12-pole, three-phase synchronous generator has 180 stator slots with a double-layer winding and eight turns per coil. The coil pitch on the stator is 12 slots. The conductors from all phase belts (or groups) in a given phase are connected in series.

(a) What flux per pole would be required to give a no-load terminal (line) voltage of 13.8 kV?

(b) What is this machine's winding factor k_w?

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